Magnetic Resonance Microscopy of Electric Currents

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Current-density imaging (CDI) is a new and encouraging achievement of MRI techniques development. The first experiments measuring electric-current distribution by MRI focused on the demonstration of the feasibility of the method on large water phantoms, where sensitivity considerations and limitations were considered. In this paper, electric currents through small conductive samples were measured. A series of 2D electric-current distributions through the selected slice with a plane resolution of 60 × 60 μm is presented. A theory for optimal sensitivity and resolution is established and compared to experimental data. Relaxation properties of the sample and diffusion effects, which influence these results significantly in the presence of large gradients, are emphasized. Finally, the power of CDI is demonstrated on a biologically relevant experiment—plant-stem electric-conductivity imaging. © 1994 Academic Press, Inc.

INTRODUCTION

Conductive samples that can be imaged by conventional MRI are appropriate material for measuring electric-current-density distribution. The physical principle of current-density imaging (CDI) by MRI is as follows: electric currents that flow through the sample change the magnetic field in the sample. A small static component Bz, usually a few μT, is added to the static magnetic field, which causes a shift in the Larmor frequency equal to γBz, where γ is the gyromagnetic ratio. From measurement of this frequency shift the change of the magnetic field can be determined. If we rotate the sample to three perpendicular spatial orientations, all components of the magnetic field can be measured. Finally, electric currents j can be calculated using Ampere's law

\[ j = \frac{1}{\mu_0} \nabla \times B, \]  \[1\]

where \( \mu_0 \) is the induction constant.

A modified spin-warp imaging sequence, where two current pulses are synchronized with the imaging sequence, can be used for measuring the frequency shift caused by electric currents (Fig. 1) (1, 2). The first electric pulse of duration \( T_1/2 \) is applied between the slice selection and π pulse, whereas the second pulse is applied symmetrically to the first one, after the π pulse, with equal strength and duration, but opposite polarity, so that both phase shifts are summed. The frequency shift is obtained by measuring the phase shift \( \phi \):

\[ \phi = \gamma B z T_c. \]  \[2\]

The phase image, \( \tilde{\phi} \), which is modulated with a period of 2π due to high gradients of \( B_z \), is calculated from an original complex image \( M_x + iM_y \) with magnetization components \( M_z, M_y \), \( \tilde{\phi} = \text{mod}(\phi, 2\pi) = \arctan(M_y/M_x) \). In most of the experimental conditions, 2D experiments on CDI give sufficient information and technically simplify time-consuming 3D CDI. Then just two phase images are needed; so, for example, if electric currents are flowing in \( x \) direction, magnetic fields \( B_x \) and \( B_z \) are obtained from phase images. They are measured in two perpendicular positions of the sample, and current density in \( x \) direction is calculated using a modified Eq. [1]:

\[ j_x = 1/\mu_0 (\partial B_z/\partial y - \partial B_y/\partial z). \]  \[3\]

In the calculation a phase image (\( \tilde{\phi} \)) can be used. In this way discontinuity in the phase image \( \phi \) can be corrected during discrete differentiation by adding or subtracting \( 2\pi \) to \( \partial B_z/\partial y (\partial B_y/\partial z) \), when the phase difference of two neighboring pixels exceeds \( \pi \).

This study was undertaken to apply CDI to the measurement of the electric conductivity of small samples. We performed sensitivity and resolution measurements on phantoms and compared them to a theoretical model which we have developed. We have shown that, in choosing optimal conditions for CDI, one must consider electric-current strength, its duration, diffusion characteristics of the system, and the strength of gradients. Finally, the successful application of CDI to plant-stem conductivity is presented, and other possible biological applications are discussed.

SENSITIVITY AND RESOLUTION LIMITATIONS: EFFECTS OF DIFFUSION

In MRI one can estimate the lower limit of a measurable signal-to-noise ratio (SNR), which depends on a variety of instrumental and intrinsic parameters (6). The lower limit of sensitivity in CDI is directly connected to SNR and arises
and Eq. [5] yield the final expression for the noise of the calculated current-density image $j_i$:

$$j_i = \frac{2N}{\mu_0 \gamma T_c SNR}. \quad [7]$$

The same expression as that in Eq. [7] can also be obtained from a simple model where the current density, $j_i$, is assumed to be uniform in a cylindrical sample. In such a sample, the magnetic field is a linear function of radius $r$ from the center of the cylinder: $B = (\mu_0/2)jr$. The lowest measurable current, current noise $\sigma_i$, is then the current that produces the magnetic field difference equal to $\sigma_i$ at a distance of one pixel size: $\sigma_B = (\mu_0/2)\sigma_i(FOV/N)$, which is equivalent to Eq. [7].

Contrary to the lower limit of CDI sensitivity, which is analogous to the lower limit of conventional MRI sensitivity, there is an upper limit of detectable electric current that does not have a direct analogy with conventional MRI, but arises from the phase-image characteristics. The problem arises if the absolute phase difference between neighboring pixels exceeds $\pi$, and then the reconstructed phase image is not predictive (5). Consequently, the maximum allowed magnetic field difference is $\Delta B = \pi/(\gamma T_c)$. Using the simple cylindrical sample model, and considering that the maximum measurable current $j_{\text{max}}$ produces the maximum allowed magnetic field difference $\Delta B$ on a distance of a pixel size $\text{FOV}/N$, the following expression is obtained:

$$j_{\text{max}} = \frac{2\pi N}{\mu_0 \gamma \text{FOV} T_c}. \quad [8]$$

The same conclusion can be also derived from the $k$-space NMR signal treatment in the presence of electric currents (2).

The signal-to-noise ratio in the current-density image, $\text{SNR}_{j}$, which can be defined as $\text{SNR}_{j} = j/\sigma_i$, obviously has an upper limit that arises from the upper limit for the measurable electric-current density $j_{\text{max}}$, that is

$$\text{SNR}_{j} \leq \text{SNR}_{j,\text{max}} = \pi/\text{SNR}. \quad [9]$$

Evidently, the signal-to-noise ratio in the current-density image can be a few times better than the signal-to-noise ratio of the conventional MR image, when conditions of maximum measurable current are approached.

The signal-intensity amplitude in MRI in spin-echo sequences decays exponentially with $T_2$. In the presence of
magnetic field gradients, signal intensity is reduced additionally because of diffusion processes. The peak signal attenuation in the middle of $k$ space, calculated for the simplified spin-warp imaging sequence in Fig. 1 using the Stejskal–Tanner formula (4) for the spin-echo experiment in the presence of the magnetic field gradients, is given by

$$\text{Signal}(T_e, \text{FOV}) \propto \text{FOV}^2 \exp\left[-\frac{(T_e + 6T)}{T_2} - K(T_e, \text{FOV})\right].$$ [10]

where $T_e$ is the total current pulse duration (echo time is then $\text{TE} = T_e + 6T$) and the field of view is FOV. The attenuation exponent $K$ is

$$K = \frac{\gamma^2 D}{g_1^2(\frac{1}{2} T_1^2) + g_2^2(\frac{1}{2} T_1^2 + \frac{1}{2} T_2^2)} + \frac{g_3^2}{(12 T_1^3 + 4 T_2^2 T_e)}.$$ [11]

Here $D$ is a diffusion constant, $T$ is a pulse-sequence-characteristic time constant defined in Fig. 1, and all active gradients are considered: readout gradient $G_r$, slice-selection gradient $G_s$, and the magnetic field gradient produced by electric currents $G_e$. The phase-encoding gradient $G_p$ lacks effect on the attenuation of the signal since the phase gradient has a zero value in the middle of $k$ space.

The image signal-to-noise ratio SNR is proportional to the signal peak in the middle of $k$ space (6), and the expression for the SNR, derived in dimensionless variables, is

$$\text{SNR}(t, v) = \frac{A v^2 \exp(-C_0 t - C_2 t)}{(1 + v^2 C_1 t - C_1^0)},$$ [12]

where $A$ is a proportionality constant, $t = T_e / T$, $v = \text{FOV} / \text{FOV}_r$, and the coefficients $C$ are

$$C_0^0 = 6 \frac{T}{T_2} + \frac{1}{2} D \gamma^2 T_3 G_1^2$$

$$C_0 = 12 D T \left(\frac{r}{T_2}\right)^2$$

$$C_1 = \frac{T}{T_2}$$

$$C_2 = \frac{1}{2} D \gamma^2 T_3 G_1^2$$

$$C_3 = \frac{1}{2} D \gamma^2 T_3 G_2^2$$

The readout gradient $G_r$ was replaced by the FOV using the relation $\gamma G_r \text{FOV} = 2 \pi N / T_s$, where signal-acquisition time $T_s = 5 T / 2$ (Fig. 1), so that $G_r = (4 \pi N) / (5 \gamma T \text{FOV})$. The reference field of view FOV was defined with boundary conditions SNR $= 1$ and $T_e = 0$:

$$\text{SNR}(0, 1) = A \exp(-C_0 - C_0^0).$$ [14]

We can now describe the behavior of noise in the electric-current image $\sigma_j$ and the maximal electric-current density that can be imaged $j_{\text{max}}$ (Eqs. [7] and [8]) in the presence of MRI conditions using a reference current density $j_r = (2N) / (r \mu_0 \gamma / \text{FOV}_r)$ and dimensionless variables $\sigma_e = \sigma_j / j_t$ and $c_{\text{max}} = j_{\text{max}} / j_t$:

$$\sigma_e = \frac{1}{tv^2} \exp\left[C_3 t^2 + C_2 t + \frac{C_1^0}{1 + v^2} + C_1^0 t - C_1^0 (1 - 1/v^2)\right]$$

$$c_{\text{max}} = \frac{1}{tv^2}.$$ [15]

Since an upper limit for measurable electric-current density exists (Eq. [8]), we can estimate an upper limit for coefficients $C_2$ and $C_3$, using the equation $G_e = (\mu_0 / 2) j$ for magnetic field gradients in a cylindrical sample caused by electric current: $G_e = \frac{1}{2} D T (\pi N / \text{FOV}_r)^2 / (v^2)$ and $C_3 = \frac{1}{2} D T (\pi N / \text{FOV}_r)^2 / (v^2)$. The terms with $C_2$ and $C_3$ have the same $r$ and $v$ dependence as the terms with $C_0^0$ and $C_1^0$, respectively; thus the upper limit for the electric-current-density noise $\sigma_e$ is

$$\sigma_e \leq \frac{1}{tv^2} \exp\left[1.13 C_1^0 / (1 - 1.33 / v^2)\right].$$ [16]

Evidently the effect of electric-current gradients on the attenuation of the signal cannot exceed the effect of the readout gradient. In practice, the electric-current density is much smaller than $j_{\text{max}}$, so that the effect of electric-current gradients on attenuation of the signal is negligible, and Eq. [16] is simplified into two exponential parts, the first term being a function of diffusion, and the second a function of $T_2$ relaxation only:

$$\sigma_e = \frac{1}{tv^2} \exp\left[4 DT \left(\frac{4 \pi N}{5 \text{FOV}_r}\right)^2 \times [(t + 3) / (v^2 - 3) + \frac{T_1}{T_2}]\right].$$ [17]

In a chosen experimental setup one can manipulate $T_e$ and FOV so that the diffusion or relaxation part of the exponent prevails. The function $DT_2 R$ can be defined as a ratio between the diffusion and the relaxation terms:

$$DT_2 R = 4 D T_2 \left(\frac{4 \pi N}{5 \text{FOV}_r}\right)^2 \left[\frac{1}{v^2 - 3} \left(1 - \frac{1}{v^2}\right)\right].$$ [18]

If $DT_2 R \gg 1$, then the diffusion effect overwhelms the relaxation effect so that the relaxation term in Eq. [17] is negligible, whereas if $DT_2 R \ll 1$, the relaxation effect overwhelms the diffusion effect, and the diffusion term in Eq. [17] can be neglected.

Sensitivity of the CDI experiment is inversely proportional to the electric-current-density noise $(1 / \sigma_e)$, and we can see
from Eq. [17] that it increases approximately as a third power of FOV, and has at fixed FOV a maximum at \( T_c \) optimum given by

\[
\frac{1}{T_c \text{ optimum}} = \frac{1}{T_2} + 4D\left(\frac{4\pi N}{5\text{FOV}}\right)^2.
\]  

This equation can provide a valuable basis for planning optimal experimental conditions.

**EXPERIMENTS ON A MODEL SYSTEM**

A sample consisting of two concentric cylinders was constructed to verify the theoretical considerations presented in the previous section. An inner cylinder of diameter 4 mm was filled with a saline solution and closed with electrodes on both sides; the outer cylinder had a diameter of 10 mm and was filled with a reference layer of water. The separation between the electrodes was 13 mm; the electrodes were connected to an amplifier of 10 V output voltage, and the salt concentration in the inner cylinder was 2%, so that the current density through the inner cylinder was \( j = 1600 \text{ A/m}^2 \) when electric pulses were on. The sample was inserted into a magnet with the axis of cylinders perpendicular to the direction of the static magnetic field. A 2D CDI experiment was performed, with the imaging plane transverse to the cylinder axis.

A series of images was recorded where all imaging conditions were the same—FOV = 15 mm, SLTH = 2 mm, TR = 1000 ms, NS = 40 (number of scans), \( T = 4 \text{ ms} \) (pulse-sequence characteristic time constant)—except the duration of the electric-current pulses \( T_c \), which was \( T_c = 3, 25, 50, 75, 100, 125 \) ms in different experiments, and TE = 23 ms + \( T_c \). Relaxation time \( T_2 = 1.5 \text{ s} \) was measured using the CPMG sequence, while a diffusion constant, \( D = 2.3 \times 10^{-9} \text{ m}^2/\text{s} \), was taken from the literature (4). The gradient strengths used for this experiment were readout gradient \( G_r = 0.04 \text{ T/m} \), slice-selection gradient \( G_s = 0.01 \text{ T/m} \), and electric-current gradient \( G_c = 0.001 \text{ T/m} \). Electric-current images were calculated without subtraction of two phase images so that \( f = \sqrt{2} \) (except the image at \( T_c = 3 \text{ ms} \), where \( f = 1 \)). From this data the coefficients in Eq. [13], \( C_0^r, C_1^r, C_2, \text{ and } C_3 \), can be calculated, whereas the coefficients \( C_0^c \) and \( C_1^c \) remain a function of the reference field of view FOV. They were calculated by fitting experimental SNR data of conventional MR images (Fig. 2, histograms) to the proposed theoretical model for SNR in Eq. [12]. The fit of experimental SNR to the theoretical model gave a value for FOV of 5.75 mm, \( j_c = 47,000 \text{ A/m}^2 \), and the coefficients \( C: C_0^r = 0.0174, C_1^r = 1.40, C_0^c = 0.00267, C_1^c = 0.467, C_2 = 2.68 \times 10^{-5}, \text{ and } C_3 = 3.57 \times 10^{-6} \).

Figure 2 shows the experimental results. The recorded images are presented as a conventional magnitude MR image, the image of the real part of the MR signal, and the calculated electric-current-density image. The SNR of magnitude images decreases with increasing \( T_c \), as can be clearly seen from the histograms. Two histogram peaks, one belonging to the background noise and the other to the phantom signal, which are very narrow in the first row image, become broader because of the loss of signal. In the bottom row image they are hardly separated.

The effect of increasing \( T_c \) can be also seen on the image of the real component of the signal. Consecutive dark and bright stripes, which are very wide at short \( T_c \), get thinner at long \( T_c \). These stripes are regions of constant phase defined by the equation \( \cos(\phi) = \text{Const} \).

The calculated current images differ significantly in SNR. This can be clearly seen from the image profile through the current region. The current image in the first row, with very short \( T_c \), has a very bad SNR, though SNR was very good. Current images in the second and the third row have the best SNR. This is because the applied \( T_c \) was approximately equal to the \( T_c \) optimum, which was 56 ms in our experiment. On images where \( T_c \) is longer than \( T_c \) optimum (Eq. [19]) (images in the fourth, fifth, and sixth row), SNR decreases again. SNR calculated from these measurements also proves that SNR can be higher than SNR (Eq. [9]).

Our model experimental setup is appropriate for the demonstration of some other interesting features of the proposed theoretical model. First, it can be shown that there is a region in the (FOV, \( T_c \)) plane where it is not possible to image any electric-current density. These regions are defined by the condition that the noise in the electric-current-density image (Eq. [7]) is greater than the maximum electric-current density that can be imaged (Eq. [8]); that is, \( \sigma_c \geq j_{\text{max}} \), or equivalently, \( \sigma_c \geq c_{\text{max}} \). Besides this, it is obvious that current
densities $j$, where $j$ is lower than the noise in the current-density image (SNR, < 1), cannot be imaged (Fig. 3, white region). For each experiment there are also regions in the (FOV, $T_2$) plane where the decay of the signal is mainly a consequence of $T_2$ relaxation, or mainly diffusion, or both. These regions are dependent on $T_2$ and $D$ (Eq. [18]). Figure 3 shows these regions calculated from conditions of our model system experiment. Experimental points taken from model-system measurements (Fig. 2) are presented as well.

It is also evident from Eqs. [7] and [8] that for each FOV and $T_2$ there is an upper and lower limit for the electric-current density that can be imaged. Figure 4 shows a plot of the region in the ($j$, $T_2$) plane where it was possible to image the electric-current density in our experiment. Experimental points taken from model-system measurements (Fig. 2) are also shown. Additional information on signal-to-noise ratio in a current-density image for the experimental conditions as they were in the experiment is shown in Fig. 5. Again, the theory shows an excellent fit with the experimental conditions.

**EXPERIMENTS ON PLANT STEMS**

Plant-stem CDI is a prime example of the capabilities of the method. The internal structure of a stem results in heterogeneous conductivity and consequently in heterogeneous current-density distribution. The stems in our experiments (*Pelargonium zonale, Rosa arvensis*) had a diameter of 8 mm. Before imaging, they were moistened in a 10% saline solution to increase their conductivity. The stems were placed in a special holder which was closed on both sides with electrodes 13 mm apart. Electric-current densities achieved in this way were approximately 1000 A/m². An experimental setup was chosen so that SNR, was maximized, and applied current pulses were $T_2 = 25$ ms long, while other measuring parameters were $TR = 1000$ ms, $TE = TC + 23$ ms, FOV = 15 mm, $SLTH = 3$ mm, and $NS = 30$.

For comparison with the model system, Fig. 6 presents a conventional MR image, an image of the real component of the signal, and the current-density image. Again, in all three cases the image of the real component of the signal has a pattern of dark and bright stripes that are attributed to the presence of electric currents. The electric-current-density image evidently shows the regions of strong conductivity that do not correlate with the bright regions on a conventional MR image. These regions can easily be distinguished from the average background noise that is attached to zero current density.

**DISCUSSION**

Density can be directly visualized by CDI due to magnetic field changes caused by an electric current passing through a material. Sensitivity and resolution of magnitudes comparable to those of conventional MRI can be achieved. Signal-to-noise ratio in the current-density image mainly depends on the duration $T_2$ and the magnitude of magnetic field $B$ caused by the applied electric-current pulses. Long current pulses with high intensity produce high phase shifts, $\phi = \gamma T_2 B$, that are a precondition for a good signal-to-noise ratio in CDI. Unfortunately, due to $T_2$ relaxation and diffusion effects, long pulses are inevitably connected to loss of the signal. The phase noise is inversely proportional to SNR; therefore the use of long pulses results in a high phase noise. These two competitive effects reach optimum conditions for CDI when the phase/phase noise ratio is maximized at (Eq. [19]).
FIG. 6. CDI experiments on two plant stems [(A) Pelargonium zonale; (B) Rosa arvensis] presented with a conventional magnitude MR image (left), the image of the real component of the MR signal (middle), and the electric-current-density image (right). Instrumental conditions were FOV = 15 mm, SLTH = 3 mm, TR = 1000 ms, NS = 30, $T_S = 25$ ms, and TE = 23 ms + $T_s = 48$ ms.

\[
\frac{1}{T_{\text{optimum}}} = \frac{1}{T_2} + 4D \left( \frac{4\pi N}{5 \text{FOV}} \right)^2.
\]

Results on a phantom sample agree well with those of the theoretical model for the sensitivity of CDI. Preliminary results on plant stems promise the possibility of imaging the pattern of current-density distribution in plant stems. Resolution of less than 60 $\mu$m and the sensitivity of 150 A/m² were achieved on the phantom sample and also on plant stems. Other future prospects of CDI can be also seen in several applications where knowing the pattern of the currents flowing through the sample could result in a better understanding of the physiological processes caused by electric currents in, for example, electrochemical cancer treatment and the electric stimulation of muscles.

REFERENCES